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Wall Shear at a Three-Dimensional Stagnation Point with a Moving Wall

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Introduction

Sears and Telionis[1] have recently treated the question of the proper generalization of Prandtl's criterion for separation in other than the classical case of two-dimensional, steady flows over fixed walls. In their considerations, the solution of Rott[2] for the flow at a two-dimensional stagnation point when the wall is moving, the case of a circular cylinder rotating about its axis normal to an oncoming stream, is used to illustrate the inadequacy of associating zero wall shear with "separation." In this flow there is a line along which the wall shear is zero without the effects usually thought to accompany separation, e.g., a rapid increase in boundary-layer thickness or any other sign of breakdown of flow of the boundary-layer type. In this regard, it is noteworthy that relative to a coordinate system with respect to which flow is steady, the surface streamlines which illuminate separation in the case of fixed walls are not interesting when the wall is moving.

From the point of view of these considerations it appears interesting to treat the more general stagnation point flow over a moving wall. The flow situation corresponds, for example, to the stagnation point on an ellipsoid of revolution rotating about one of its axes normal to the oncoming stream. In the case of three-dimensional separation over fixed surfaces we consider a surface line which is the locus of tangents of surface streamlines from regions upstream and downstream (in a sense normal to the line) of separation.[3] The vector representing the surface shear is tangent to this line. As suggested above, if the wall is moving, the surface streamlines are not interesting, whereas the line along which the wall shear is tangent is significant.

Received September 10, 1973. The author gratefully acknowledges helpful suggestions by W. R. Sears.

Index categories: Fluid Dynamics; Boundary Layers and Convection Heat Transfer—Laminar.

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Table 1 Wall values for several values of the parameter c

c	$f''(0)$	$g''(0)$	$F'(0)$	$G'(0)$
1	1.3119	1.3119	−0.9387	−0.9387
0.5	1.2669	0.9981	−0.8690	−0.7637
0	1.2326	0.5705	−0.8113	−0.5705
−0.5	1.2302	−0.1115	−0.8044	−0.5115

Before proceeding we note that Rott pointed out the possibility of generalizing his solution to the case of the three-dimensional stagnation point but apparently this possibility has not been exploited.† We also note that Davey[4] provides a general analysis of the three-dimensional stagnation point in a fluid which is advancing toward the body and is rotating about the same axis as is the body.

Analysis

Our analysis is an extension of Davey[5] and Libby[6] and our notation is thus standard. The surface is the xy-plane; the external flow is characterized by

$$u_e = \alpha x \qquad v_e = c\alpha y \tag{1}$$

where the coordinate system is selected so that $c \leq 1$. The classic, well-known cases of stagnation point flow, i.e., the axisymmetric and two-dimensional cases, correspond respectively to $c = 1$ and $c = 0$. The surface ($z \equiv 0$) is considered to be moving with velocity components u_w, v_w.

The similarity variable for the general, three-dimensional case is

$$\eta = z(\alpha/\nu)^{1/2} \tag{2}$$

We assume the following form for the velocity components tangent to the surface

$$u = u_w F(\eta) + \alpha x f'(\eta) \qquad v = v_w G(\eta) + c\alpha x g'(\eta) \tag{3}$$

The continuity equation supplemented by the inessential but convenient assumption that $f(0) = g(0) = 0$ leads to the third velocity component

$$w = -(\alpha\nu)^{1/2}(f + cg) \tag{4}$$

Substitution of Eqs. (3) and (4) into the x-wise and y-wise momentum equations and collection of powers of x, i.e., x^0, and x, leads to the equations for $F(\eta)$ and $G(\eta)$

$$F'' + (f + cg)F' - f'F = 0 \qquad G'' + (f + cg)G' - cg'F = 0 \tag{5}$$

and for $f(\eta)$ and $g(\eta)$

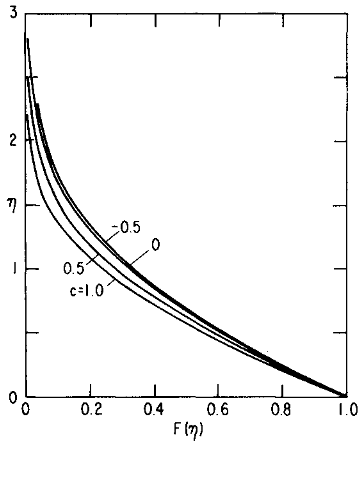

Fig. 1 The normalized x-wise velocity profiles for various values of the parameter c.

† The author is indebted to N. Rott and A. Davey for trying to recall previous publication of the present solutions.

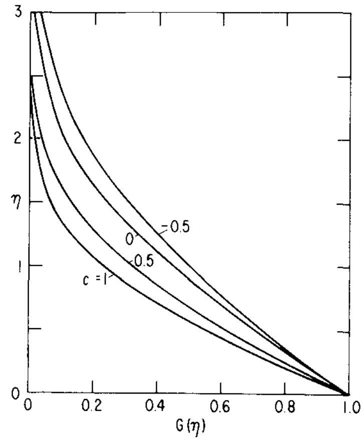

Fig. 2 The normalized y-wise velocity profiles for various values of the parameter c.

$$
\begin{aligned}
f''' + (f + cg)f'' + (1 - f'^2) &= 0 \\
g''' + (f + cg)g'' + c(1 - g'^2) &= 0
\end{aligned} \tag{6}
$$

The appropriate boundary conditions are

$$\text{at } \eta = 0: \quad f = g = f' = g' = 0, \quad F = G = 1$$

$$\text{at } \eta \to \infty: \quad f' = g' = 1, \quad F = G = 0$$

Equations (6) depend only on the parameter c and have been solved numerically by Davey[5] for a range of values of that parameter. Rott[2] provides for the case $c = 0$ the solution to Equations (5); in particular he finds $F'(0) = -0.811$. We give in Table 1 the crucial wall values, $F'(0)$ and $G'(0)$, for several values of c, and show in Figs. 1 and 2 the corresponding profiles, $F(\eta)$ and $G(\eta)$.‡ These results have been obtained by straightforward numerical means.

The values $F(0)$, $G(0)$ permit the description of the wall shear. Consider a line $y_{ws}(x)$ such that the wall shear is everywhere tangent thereto. Then y_{ws} corresponds to a solution of the equation

$$\frac{dy_{ws}}{dx} = \frac{\tau_y}{\tau_x} = \frac{v_w G'(0) + c\alpha y_{ws} g''(0)}{u_w F'(0) + \alpha x f''(0)} \tag{7}$$

Let $Y \equiv \alpha y_{ws}/u_w$, $\xi \equiv \alpha x/u_w$, $\alpha \equiv -F'(0)/cg''(0)$, $\beta \equiv f''(0)/cg''(0)$, and $\gamma \equiv -v_w G'(0)/u_w g''(0)$. The solution of Eq. (7) is readily found to be

$$Y - \gamma = C(\beta\xi - \alpha)^{1/\beta} \tag{8}$$

where C is a constant of integration which may be conveniently evaluated by specifying a particular line on the surface, e.g., the value of Y at $\xi = 0$. The line $\beta\xi - \alpha = 0$ is clearly important.

Consider, for clarity, a special case in which $c > 0$ and the surface is moving in only the x-direction so that $\gamma = 0$ and $u_w > 0$. For $\xi < 0$ there is "reverse" flow in that $u > 0$ for some region close to the wall and $u < 0$ beyond that region. The aforementioned line corresponding to $\xi = \alpha/\beta = -F'(0)/f''(0)$ further divides the surface into two regions; to the left of this line $\xi < \alpha/\beta$, the shear on the wall is to the right, while to the right, $\xi > \alpha/\beta$, this same shear is to the left. Thus the line $\xi = \alpha/\beta$ is a locus of surface shear lines dividing the flow into "upstream" and "downstream" regions and is analogous to a three-dimensional separation line. However, it does not have associated with it the other manifestations of "separation." We show schematically in Fig. 3 some of the features of this special case.

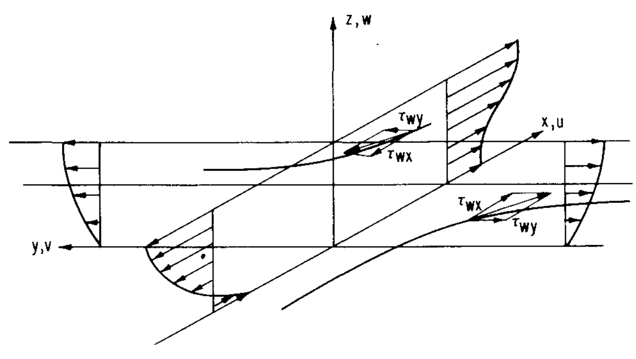

Fig. 3 Schematic representation of the stagnation point boundary layer with the wall moving in the x-direction.

We have thus shown the three-dimensional analog of Rott's two-dimensional separation. The above solution for the three-dimensional stagnation point over a moving wall can be generalized to the compressible case, but we do not do so here.

‡ Because they do not appear to be relevant for present purposes we do not consider the "discontinuous branch" solutions for $c < 0$, cf. Ref. 6.

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Incompressible Potential Flow Solutions for Arbitrary Bodies of Revolution

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I. Introduction

THE potential flow about an arbitrary body of revolution was first treated by Theodore von Kármán.[1] He determined the potential flow around bodies of revolution at zero angle of attack by superposing a uniform stream on a system of line sources distributed along the axis of the body. The strengths of the sources were determined so that the zero streamline passed through given coordinates of the body, the number of coordinates being equal to the number of sources.

Since the original work of von Kármán, very little enlightening

Received September 25, 1973.

Index categories: Hydrodynamics; Aircraft Aerodynamics (Including Component Aerodynamics).

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